Computer algebra systems: Permitted but are they used?

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Introduction

Since the 1990s, computer algebra systems (CAS) have been available in Australia as hand-held devices designed for students with the expectation that they will be used in the mathematics classroom. Prior to the development of hand-held CAS (first the TI-92 in 1995), these programs (for example: *Derive*, *Mathematica*, *Maple*) were available for computers but had limited penetration in schools. In Victoria, CAS use at school and at home was permitted for senior secondary mathematics students completing externally set extended tasks known as 'Common Assessment Tasks' (Leigh-Lancaster & Rowe, 1999). However, in 2002 further institutional value was accorded to the use of CAS through the trial of a new senior secondary mathematics subject: Mathematics Methods (CAS). For this subject CAS use was also permitted in the final, high stakes, externally set examinations. After more than a decade with CAS readily available and its use encouraged for teaching, learning and assessment, this paper considers the questions of

1. To what extent has CAS use been taken up by teachers as part of their classroom practice? and
2. How much use are students making of CAS?

From a wider survey on students' perception of the use of technology for teaching and learning mathematics (656 students) we have extracted the responses from those who said that they used CAS in their year 12 examinations (334 students), thus assuming that for these students, 'using technology for mathematics' equates to 'using CAS'. In the sections below we outline the potential pedagogical opportunities technologies like CAS offer and some possible constraints to taking up such opportunities. Next we provide details, including results, for the study reported in this paper. This is then followed by some discussion and conclusions.

Literature review: Teaching with CAS

Opportunities

Pierce and Stacey (2010) summarised the pedagogical opportunities of mathematics analysis software—of which CAS is the prime example—in the schematic map reproduced in Figure 1. This map summarises the findings of many researchers' work from the previous twenty years including, for example, the work of Heid (1998), Lagrange, Artigue, Laborde and Trouche (2003), and Kieran and Damboise (2007). The map draws attention to pedagogical opportunities, afforded by the speed and accuracy of CAS, for the teachers to set tasks that foster higher or broader mathematical thinking. Such tasks may otherwise be too time consuming or difficult for some students. This thinking may be facilitated by the various display modes (graphical, tabular, algebraic, etc.) that now even hand-held CAS provide simultaneously or sequentially. Such easy access to a suite of mathematical representations means that CAS can be used by students as an available authority: to support mathematical exploration by varying parameters; studying variations; making and testing generalisations; and working in a range of representations in order to highlight mathematical points or progress towards the solution of mathematical problems. While such use of CAS is possible and can make it

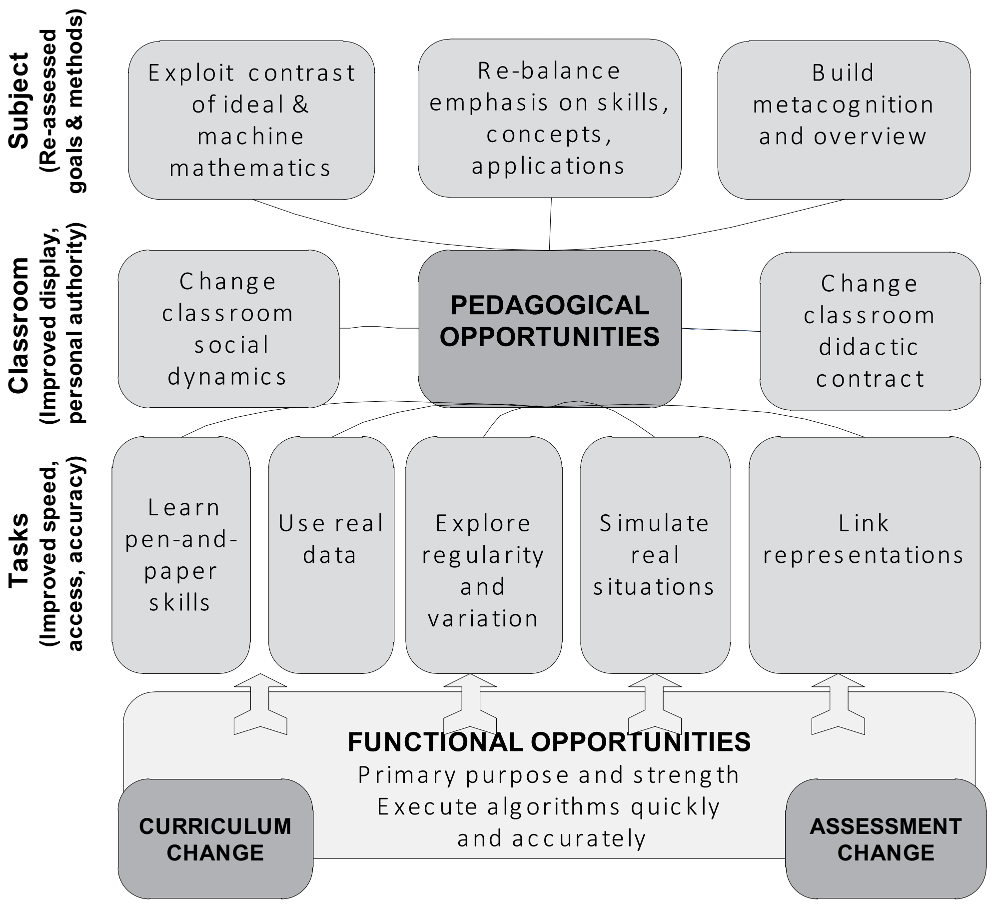

Figure 1. Pedagogical map for mathematics analysis software (Pierce & Stacey, 2010, p. 6).

a powerful tool for both doing and learning mathematics, teaching students to use CAS in these ways, that is to make effective use of CAS (see Pierce & Stacey, 2004), may result in change in classroom expectations, in the 'didactic contract' (Brousseau, 1998) and in the social dynamics of the classroom as CAS effectively becomes an active contributor and students are empowered to solve problems using a range of different approaches. Finally, permitting the use of CAS in the classroom and in assessments opens discussion among teachers and curriculum developers about what aspects of mathematics could now be taught and how.

Constraints

The use of CAS for teaching and learning will depend not only on its availability but on teachers' beliefs about mathematics and their attitude towards CAS as a tool for students and as another 'player' in the classroom. In their study of a sample of Victorian teachers, Forgasz and Griffith (2006) found that teachers were generally optimistic about the effects of CAS calculators in teaching, student learning and the curriculum. Like Pierce and Ball (2009), they found some dissenting voices particularly among the 35-45 age range; mid-career teachers. Pierce and Ball (2009) report on a questionnaire targeting teachers' use of technology, including CAS. Almost a quarter of Pierce and Ball's 92 survey respondents agreed or strongly agreed that "students don't understand maths unless they do it by hand first". This has been a key issue impacting on their use, or not, of CAS. Leigh-Lancaster (2010) also notes that students' competence with traditional 'by hand' (i.e., pen and paper) skills is raised perennially as a concern by some mathematics teachers. Many anticipate that there will be a negative impact on the development of these skills if CAS use results in less practice of pen-and-paper ('by hand') calculations and manipulations. This issue was monitored carefully by the VCAA from 2006–2009 when only part of the cohort (7000+ students in 2009) studied the CAS version of the Mathematics Methods subject and one final examination was 'technology free' and one 'technology active'. Students' performance for matched questions on the technology-free examination consistently showed similar results for the CAS and non-CAS cohorts with any differences being in favour of the Mathematics Methods (CAS) students. This pattern persists when data is controlled for general mathematical ability.

Özgün-Koca (2010) notes that, despite their availability, CAS calculators have not impacted the study of symbolic algebra to the extent that graphics calculators have impacted the study of graphs. She also conjectures that while teachers place importance on pen-and-paper skills for symbolic manipulation, they appreciate the speed and accuracy of a function grapher. The main results of Özgün-Koca's study of pre-service teachers showed that, prior to demonstration of the possible uses of CAS, a majority (59%) of these prospective teachers did not believe that CAS calculators would be beneficial for algebra instruction. Özgün-Koca conjectures that greater awareness of

teachers' views and uncertainties could inform the provision of information in teacher education classes.

As a result of a US study, Quesada and Dunlap (2011) suggest that pre-service and in-service teachers' own lack of experience of working mathematically with technology such as CAS is a barrier to their pedagogical use of these tools. They propose a range of possible new concepts and approaches to pre-calculus topics fostered by hand-held graphic technology (such technology is similar to CAS calculators but lacks a symbolic algebra module). They tested pre-service and in-service mathematics teachers from mid-west USA using mathematical problems best solved with the aid of such technology. The mean and standard deviation test scores for in-service teachers were 54.3% and 23.9%, respectively, and for pre-service teachers 46% and 18.5%. The self-ratings on their use of technology in teaching or students' learning content respectively were (on a Likert scale from *1: Not at all* through *3: Some* to *5: Continuously*) 2.76 and 2.21. Highest use was for considering intervals where a function is increasing or decreasing. The researchers question the lack of exposure that these teachers and pre-service teachers have had to both doing mathematics and teaching with technology. They suggest the need to improve the preparation of pre-service teachers for the proper integration of technology.

Lagrange (2007) reminds the reader that innovations, such as CAS, are adopted by teachers if they are seen to provide an answer to a perceived issue. If teachers are comfortable with their current teaching strategies then they will be reluctant to change, especially when that change involves personal and classroom time to become familiar with the technology and risks since the lessons may not follow their familiar path. He notes that teachers are concerned that they will lose 'didactic time' because they perceive that the time needed to learn the technology will not be repaid by more efficient or intense mathematical teaching time. This was evident in Pierce and Ball's (2009) survey, when almost a quarter of the respondents agreed or strongly agreed with the statement 'if I use more technology I won't have time to cover the course'. Lagrange notes that technology supports new mathematical techniques but in doing so may make well tried teaching practices obsolete. He comments that reconstructing their practice is difficult for many teachers and the pace of change with which each teacher can cope, varies.

This study

Context

The data discussed in this paper was collected as part of a pilot study that investigated first year university mathematics and statistics students' understanding of functions and variables, as well as the use of technology in their last year of school (Year 12). Students' understanding of the notions of functions and variables have been detailed elsewhere. (Interested readers

should see Bardini, Pierce, Vincent & King, 2014 and Bardini, Vincent, Pierce & King, 2014 for further details). In this paper we focus on the usage of technology reported by students. The data was collected at a leading Australian university where the students are likely to have been highly successful secondary school students. Data was collected during Semester 1 from first year mathematics students using a voluntary, online survey made available via the university's learning management system. The students would normally access this system several times each week and, in addition, the students' lecturers drew their attention to the survey.

Of the approximately 2000 students who were given access to this survey, some 656 responded. From this we selected those respondents who had studied Year 12 (final year of secondary schooling) in Victoria and who indicated that they had used CAS in their final examination. This reduced the pool of data to 334 students but meant that we knew that they had studied in an education jurisdiction where CAS use is encouraged and its availability is assumed in one of the final examinations. It also meant that we had access to the relevant curriculum and assessment guidelines (VCAA, 2010).

Framework and method

The items related to technology use (in this case CAS), discussed in this paper, were framed by the 'Task' level of Pierce and Stacey's map of the pedagogical opportunities offered by mathematics analysis software like CAS (see Figure 1). There were no items focusing on the Classroom and Subject sections of the framework (middle and top rows of the map) because noting adoption of these opportunities requires comparison to either typical or previous teaching. Students are not normally in a position to identify and comment on these aspects of teaching.

The survey first asked the students basic demographic questions including where they had undertaken their final year of schooling. They were then presented with two parallel sets of twelve items related to the use of technology for mathematics. We are taking 'technology' to be CAS since we have selected the responses from Victorian students who said that they used CAS in the classroom and in their final examinations. The first set of items focused on the students' perceptions of their teachers' use of technology in the classroom. The 'classroom' restriction was made because we are interested in the pedagogical use of CAS that would be observed by the students. Students were asked: "For the next 12 items, think about YOUR TEACHER'S USE, in class, of mathematics technology. Did they use it for:" The second set focused on the students' own use of CAS, inside and outside the classroom: "For the next 12 items, think about YOUR USE of mathematics technology. Did you use it for:" The options for both sets were: "Not used", "Hardly used", "Occasionally in a few topics", "Occasionally in most topics", "Often in a few topics", "Often in most topics".

The wordings for the different categories of use of technology (in this case CAS) given in the survey are shown in Table 1 with their abbreviations (used for the purpose of this paper) in the right hand column.

Table 1. Words used in survey items and their abbreviations.

Words used	Abbreviations
Checking answers	Checking
Obtaining results faster than with pen & paper	Faster
Doing calculations that students might find hard	Hard Calculations
Doing algebra that students might find hard	Hard Algebra
Doing application problems	Applications
Showing the impact of varying coefficients, powers etc.	Varying
Creating tables	Tables
Graphing functions	Graphing
Solving	Solving
Expanding or factorising	Expand /Factor
Differentiating or integrating	Diff/ Integrate
Doing matrix operations	Matrices

Results

The survey results are summarised as percentages, in Table 2, to indicate the levels of perceived use and with side-by-side bar graphs, in Figures 2a–2l, to visually illustrate the difference in patterns of use by teachers and their students. In Table 2 percentages may add to more than 100% since the items allowed multiple responses. It seems logical that some features of CAS might, for example, be used both often in a few topics and occasionally in a few topics. As a general observation we can see that, as far as the students perceived and

Table 2. Percentage of students reporting perceived use of CAS by their mathematics teacher and themselves.

Level of use	Not used		Hardly used		Occasionally in a few topics		Occasionally in most topics		Often in a few topics		Often in most topics	
PURPOSE Teacher(T)/ Student(S)	T	S	T	S	T	S	T	S	T	S	T	S
Checking	6.7	2.9	19.9	7.1	26.3	16.2	22.4	22.4	9.0	16.8	15.7	34.7
Faster	10.8	2.1	24.9	13.1	26.6	19.5	17.3	22.9	8.2	17.7	12.3	24.7
Hard calculations	13.9	3.0	29.1	7.9	28.5	25.0	13.6	24.4	7.3	14.6	7.6	25.0
Hard algebra	24.3	4.6	31.5	21.9	21.6	26.1	11.1	17.3	6.0	13.7	5.4	16.4
Applications	8.7	1.5	18.5	9.4	27.8	20.9	17.9	20.9	11.6	21.5	15.5	25.8
Varying	15.9	6.2	22.0	20.9	28.4	24.9	13.1	17.5	10.4	16.0	10.1	14.5
Tables	24.2	22.4	26.6	33.5	26.6	19.3	6.4	9.7	11.6	10.9	4.6	4.2
Graphing	2.4	1.2	6.9	1.2	17.7	13.7	17.4	18.1	26.4	21.8	29.1	43.9
Solving	7.9	2.4	20.8	11.9	26.7	21.7	15.7	21.4	13.8	20.8	17.9	21.7
Expand/Factor	19.2	7.1	34.4	23.8	22.5	28.9	12.9	16.4	5.7	14.6	5.4	9.2
Diff/Integrate	11.1	2.7	25.2	11.7	26.4	26.5	13.2	19.3	13.8	25.3	10.2	14.5
Matrices	8.2	5.5	22.5	15.0	29.8	28.8	8.2	11.3	18.8	24.8	12.5	14.4

Note: These were multiple response items so totals may add to more than 100%.

recalled, most teachers made very limited use of CAS and students made far more use of CAS than did their teachers (Figures 2a–2l). We will now look at the detail.

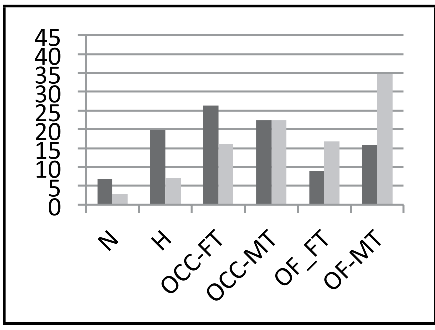

Figure 2a. Checking.

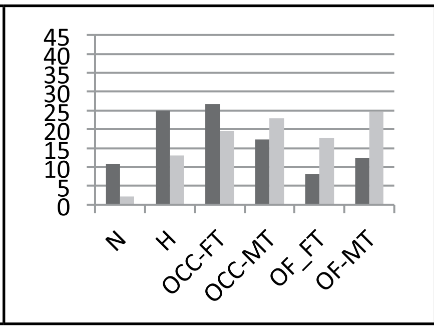

Figure 2b. Faster.

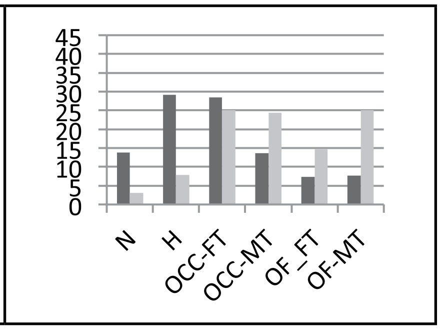

Figure 2c. Hard Calculations.

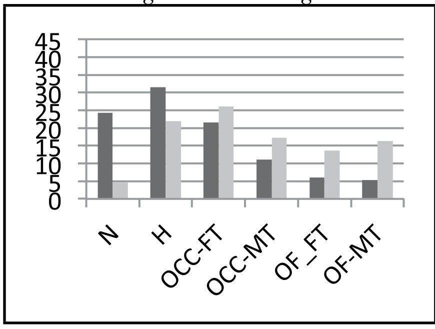

Figure 2d. Hard Algebra.

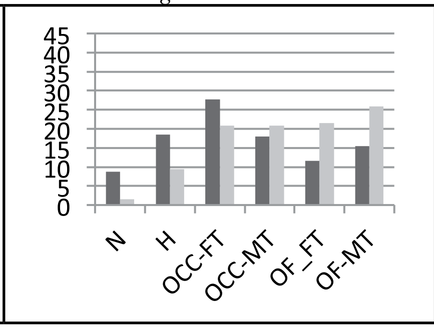

Figure 2e. Applications.

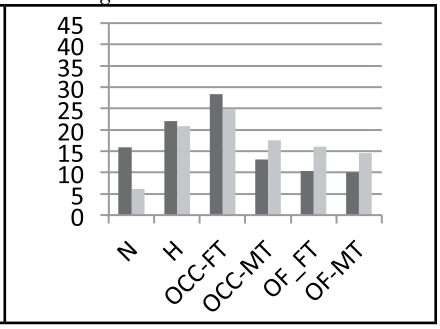

Figure 2f. Varying.

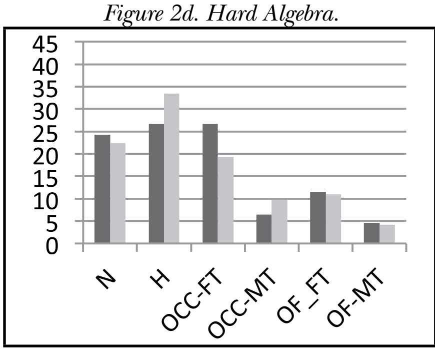

Figure 2g. Tables.

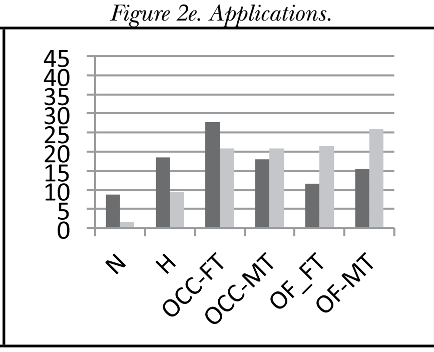

Figure 2h. Graphing.

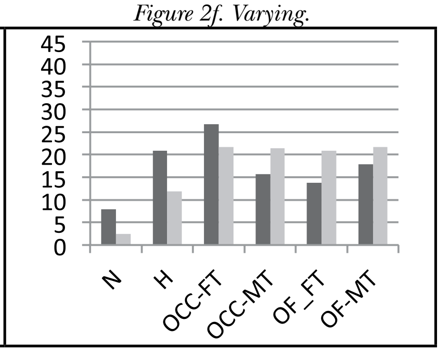

Figure 2i. Solving.

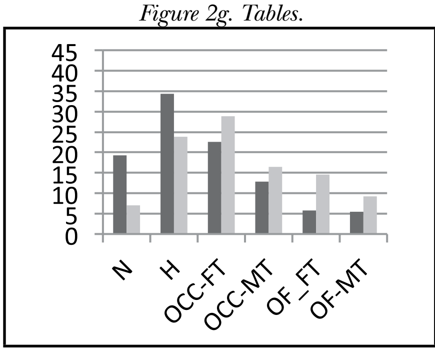

Figure 2j. Expand/Factor.

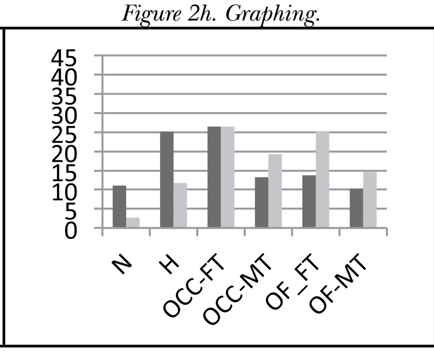

Figure 2k. Diff/Integrate.

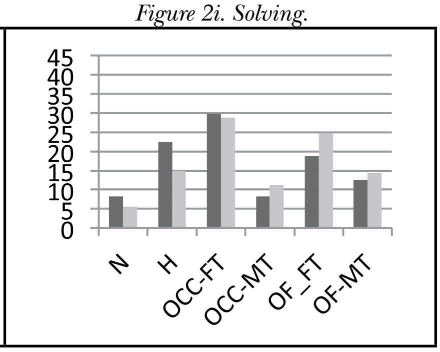

Figure2l. Matrices.

N= not used; H= hardly used; OCC-FT = occasionally used in a few topics;
OCC-MT = occasionally used in most topics; OF-FT = often used in a few topics;
OF-MT= often used in most topics

Teachers Students

Figures 2a–2l. Percentage of students perceiving use of CAS for varying purposes.

Students recalled that their Year 12 mathematics teachers had used CAS most for Graphing (Table 2: "Often in a few topics" 26.4%, and "Often in most topics" 29.1%, with a total of only 9.3% in the "Not used" or "Hardly used" categories). On the other hand, they perceived that their teachers made least use of CAS to do Hard Algebra ("Not used" 24.3%, "Hardly used" 31.5%).

Students showed a different pattern of use. In general they perceived that they made more use of CAS, more often, in more topics, than did their teachers (Figures 2a-2l). Like their teachers, most use of CAS was for Graphing (Figure 2h) and most students also used their CAS for: Checking, Faster, Hard Calculations, and Applications (Figures 2a, 2b, 2c, 2e). Interestingly less use was made of this technology to support Hard Algebra than Hard Calculations (Table 2).

Discussion and conclusions

The use of CAS in the mathematics classroom is an innovation that began more than twenty years ago. The sociologist Rogers (1995) reiterates a scheme he proposed in 1962 to describe the diffusion of innovations in society. He suggested that innovations are first taken up by a few innovators and then enthusiastic early adopters, eager to explore the capabilities of a new artefact. We find reports in the literature of innovators and early adopters of CAS in the classroom (see for example: Jakucyn & Kerr, 2002; Edwards, 2003; Neill & Maguire, 2006). Following these groups is the early majority, those who accept the necessity for change but wish to move forward carefully and thoughtfully, learning from the experience of the early adopters rather than paving the way. Pierce and Stacey (2013) provide case studies of four such teachers learning to teach with CAS. The late majority and lastly the laggards follow in adopting the innovation. Rogers linked the likely proportions of people, in this case teachers, to the normal bell curve; estimating that innovators might be 2.5% of the teachers and early adopters 13.5%, with 34% in the early majority making up half of the target group. The late majority add another 34% and the final 16% would be laggards.

From the results of our study we see that at one extreme a few teachers were perceived to use CAS often in most topics. These teachers may be Rogers' innovators. It might be anticipated that effective use of CAS to support teaching and learning would see CAS used modelled by the teacher occasionally in most topics and often in a few topics. However, this pattern of use is evident for only a small percentage of the students' teachers. Perhaps these teachers fit Rogers' notion of early adopters. Most teachers apparently made very little use of CAS. This group fits Rogers' notion of the late majority while the few who did not use it at all may be laggards. The patterns of perceived teacher use of technology (CAS) are skewed towards non-use rather than symmetrical and seem to be missing a large group who could take further advantage of CAS,

that is, Rogers' early majority. These observations suggest that the diffusion of this innovation has either stalled or is progressing very slowly.

It appears that while teachers made use of CAS for Graphing (Figure 2h), they made little use of CAS for Tables (Figure 2g) or Hard Algebra (Figure 2d). From this result we assume that few teachers took the opportunity to link representations (far right box on the Task level of the Pedagogical opportunities map, Figure 1). It also appears that teachers made little use of CAS to support students' learning of pen-and paper skills in this way or by varying the value of parameters (Figure 2f).

Did their teachers discourage the use of CAS for algebra? Did the students actually learn how to use CAS to support their work in algebra or to support their learning of algebra? Did they find that, given the level of algebra, it was faster to work with pen-and-paper than to correctly enter algebraic expressions? This does not seem a likely explanation since quite a high percentage of students used their CAS for calculus where entry of expressions requires a similar level of understanding of algebraic structure and knowledge of the various modes and templates available in the software.

Without suitable modelling of CAS use by their teachers, students are unlikely to develop effective use of CAS (see Pierce & Stacey, 2004). The results, graphed in Figures 2a-2l, above, suggest that, while students made considerably more use of CAS than their teachers, most use was for Checking and Graphing. They made much less use for Varying, which is a strategy that promotes mathematical thinking and understanding.

The results reported in this paper are based on items included in a pilot survey. They raise questions rather than provide answers. The results do however tell us that, at least from these first year university students' recollection of their Year 12 experience, most or their VCE mathematics teachers made little use of CAS as a pedagogical tool in their classes, despite the institutional approval and encouragement indicated by both the State's curriculum and assessment for the past decade. It appears that teachers do not model effective use of CAS for their students. Further investigation is required to identify why this is so. Informal conversations suggest that, while CAS use is permitted in most of the final year assessment, teachers are most concerned about the technology –free examination. If this is the reason for their lack of teaching with CAS then it suggests that they do not perceive the pedagogical opportunities afforded by the use of CAS. A better understanding of the barriers to teachers using CAS technology to enhance their pedagogy is needed and then perhaps more effective professional learning programs can be provided for teachers.

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